CIVIL ENGINEERING SECTION – MASTER SEMESTERS 1 / 3 – 2025-2026

GEOMECHANICS

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Exercise 4 – 16.10.2025

Critical state concept

Problem statement

The exercise deals with the characterization of the critical state reached in drained and undrained CTC tests.

The following information is given:

(i) the initial condition of a clay specimen in terms of initial mean effective stress (p'_0) and initial specific volume (v_0)

(ii) the parameters of the normal compression line (NCL) and of the critical state line (CSL)

(iii) the condition in which the triaxial test is performed (i.e. drained or undrained)

The normal compression line and the critical state line of the considered clay are characterized by the following parameters:

$$N = 2.95,\ \lambda = 0.15,\ \Gamma = 2.75,\ M = 0.85$$

The clay specimen is isotropically compressed in drained condition (isotropic compression stage) up to $p'_0 = 300kPa$, the specific volume at the end of the isotropic compression is $v_0 = 2.09$. The initial pore water pressure ($p_{w,0}$) is approximately equal to 0. Then, the specimen is subjected to the shearing phase. Consider the shearing phase performed in <u>drained condition</u> for Question 1 and <u>undrained condition</u> for Question 2.

Question 1 – Drained CTC test

1. Sketch the critical state line and the stress path followed during the shearing stage of a CTC **drained** test in the planes $(q - p')$ and $(v - \ln{(p')})$.

- Calculate the corresponding values of deviatoric stress (q), mean total stress (p), mean effective stress (p'), specific volume (ν), volumetric strain (ε_{vol}) at the critical state.
- How does the specific volume at the critical state (ν_{cs}) change with respect to the specific volume at the end of the isotropic compression (ν_0) for an overconsolidated (OC) clay during the shearing stage of a CTC **drained test**. What is the physical meaning of this behaviour?

Question 2 – Undrained CTC test

- Sketch the critical state line and the stress path followed during the shearing stage of a CTC **undrained** test in the planes $(q - p')$ and $(v - \ln{(p')})$.
- Comment on the difference between the total stress path and the effective stress path during the shearing stage of a CTC **undrained** test. How is it different from the **drained** case?
- Calculate the corresponding values of deviatoric stress (q), mean total stress (p), mean effective stress (p'), specific volume (ν), volumetric strain (ε_{vol}), pore water pressure (Δp_w) at the critical state.

Which is the constant volume/critical state shear strength angle φ'_{cv} of the analysed soil?

- Does the critical state line depend on the initial void ratio?

Definitions of interest

The critical state line (CSL) is defined by the following two equations:

$q = Mp'$ in $(q - p')$ plane (1)

$v = \Gamma - \lambda \ln p'$ in $(v - \ln(p'))$ plane (2)

On the other hand, the volumetric strain can be written as follow (see annex):

$\varepsilon_v = -\frac{\Delta v}{v_0}$ (3)

in which v is the specific volume

Question 1 - Drained CTC test

- Figure A shows the stress path followed during the shearing stage of the drained CTC test in the planes $(q - p')$ and $(v - \ln(p'))$. When the shearing phase takes place in drained conditions there is no generation of pore water pressure, therefore p'=p (if the initial pore water pressure is equal to zero).

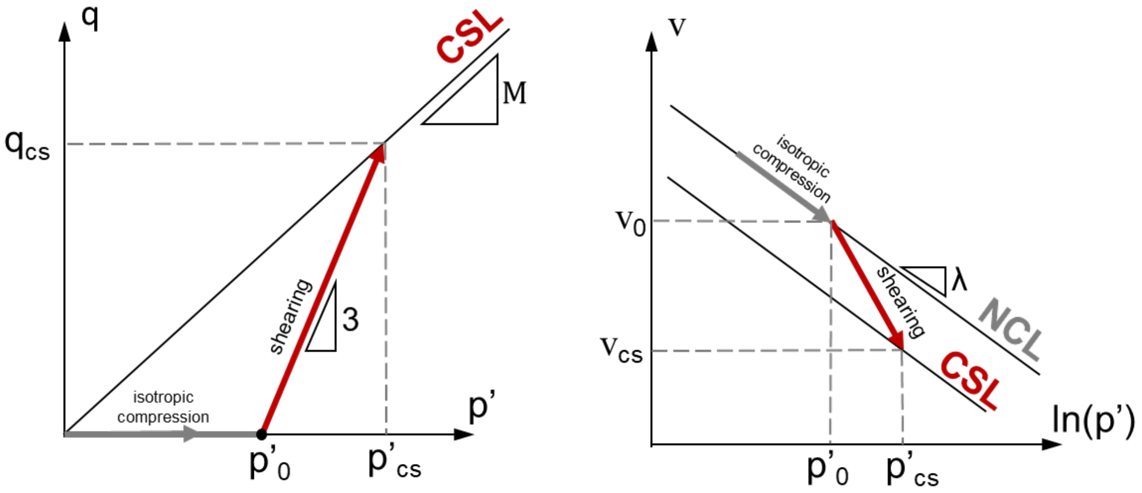


Figure A. Stress path (red line) followed during the shearing stage of the drained CTC test

- Calculation of the deviatoric stress at the critical state q_{cs}:

$$q_{cs} = Mp'_{cs}$$

q_{cs} and p'_{cs} are two unknowns, so we need a second equation that is given by the stress path:

$$q_{cs} = 3(p'_{cs} - p'_0) \quad \rightarrow \quad p'_{cs} = \frac{q_{cs}}{3} + p'_0$$

So, we can write:

$$q_{cs} = Mp'_0\left(\frac{3}{3-M}\right) = 0.85 \cdot 300\left(\frac{3}{3-0.85}\right) = 355.81\ \text{kPa}$$

Calculation of the mean effective stress at critical state using the equation of the CSL in the (q - p') plane:

$$p'_{cs} = \frac{q_{cs}}{M} = \frac{355.81}{0.85} = 418.60\ \text{kPa}$$

Calculation of the specific volume at critical state using the equation of the CSL in the (ν - ln(p')) plane:

$$v_{cs} = \Gamma - \lambda \cdot \ln(p'_{cs}) = 2.75 - 0.15 \cdot \ln(418.60) = 1.84$$

Calculation of volumetric strain at critical state:

$$\varepsilon_{vol} = -\frac{(v_{cs} - v_0)}{v_0} = -\frac{1.84 - 2.09}{2.09} = 0.12$$

- If the soil is a highly overconsolidated (OC) clay the specific volume at the critical state (ν_{cs}) will be higher than the specific volume at the end of the isotropic compression (ν_0) during the shearing stage of a CTC **drained test**. Therefore, it is said that in this case, the CSL is reached with a dilatant behaviour (volume expansion). If the soil is slightly overconsolidated a volume decrease during the shearing phase could be expected.

Question 2 – Undrained CTC test

- Figure B shows the stress path followed during the shearing stage of the undrained CTC test in the planes $(q - p')$ and (υ - ln(p')). When the shearing phase takes place in undrained conditions there is no water flow in or out of the specimen. As a consequence, an induced pore water overpressure is expected. Then, the total stress path is different with respect to the effective stress path and p'≠p.

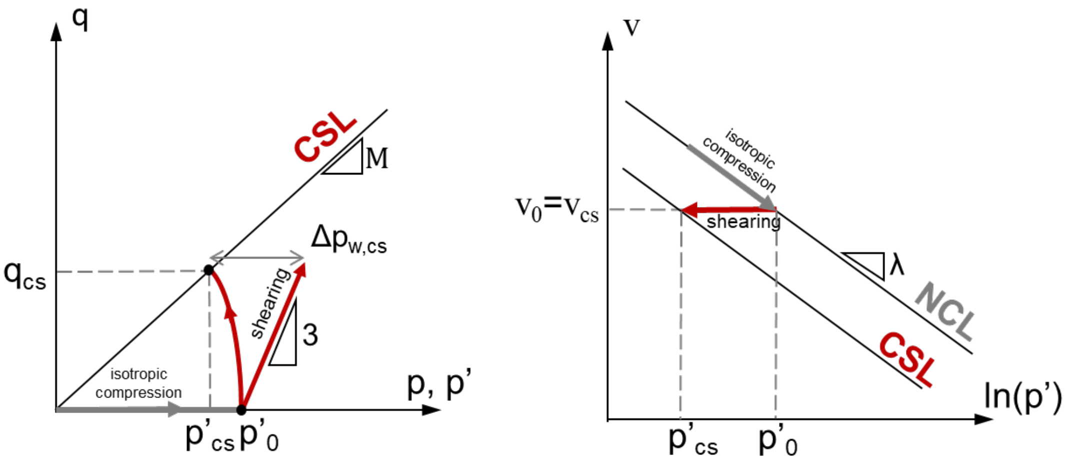


Figure B. Stress path (red line) followed during the shearing stage of the **undrained** CTC test

- When the shearing phase takes place in undrained conditions there is no water flow in or out of the specimen. As a consequence, an induced pore water overpressure is expected. Then, the total stress path is different with respect to the effective stress path and $p' \neq p$. The critical state line is the same in drained and undrained conditions for a given geomaterial. What changes is the way it is reached.

- In undrained condition, the volumetric deformation is zero; therefore, the specific volume at critical state (ν_{cs}) is equal to its initial value (ν_0). → $\nu_{cs} = \nu_0$ and $\varepsilon_{vol}=0$

Then, using the equation of the CSL in the plane (υ - ln(p')), it is possible to compute the mean effective stress at critical state (p'_{cs}).

$$\mathrm{p'}_{\mathrm{cs}} = \exp\frac{(-\upsilon_{\mathrm{cs}} + \Gamma)}{\lambda} = \exp\frac{(-\upsilon_0 + \Gamma)}{\lambda} = \exp\frac{(-2.09 + 2.75)}{0.15} = 81.45\ \mathrm{kPa}$$

Calculation of the deviatoric stress at critical state (q_f) using the equation of the CSL in the (q - p') plane:

$$\mathrm{q_{cs}} = \mathrm{M} \cdot \mathrm{p'_{cs}} = 0.85 \cdot 81.45 = 69.23\ \mathrm{kPa}$$

Calculation of the mean total stress at critical state (p_f) using the equation of the total stress path:

$$\mathrm{q_{cs}} = 3(\mathrm{p_{cs}} - \mathrm{p'_0})$$

$$\mathrm{p_{cs}} = \frac{\mathrm{q_{cs}}}{3} + \mathrm{p'}_0 = \frac{69.23}{3} + 300 = 323.08\ \mathrm{kPa}$$

Calculation of the pore water pressure at critical state ($\Delta p_{w,cs}$):

$$\Delta \mathrm{p}_{\mathrm{w},cs} = \mathrm{p_{cs}} - \mathrm{p'}_{\mathrm{cs}} = 323.08 - 81.45 = 241.63\ \mathrm{kPa}$$

Calculation of the constant volume/critical state shear strength angle φ'_{cv} of the analysed soil:

The Critical State Line (CSL) is given by the equation:

$\tau = \sigma'_n tan\varphi'_{cv}$ in $(\tau - \sigma'_n)$ plane

From the $(q - p')$ plane to the $(\tau - \sigma'_n)$ plane:

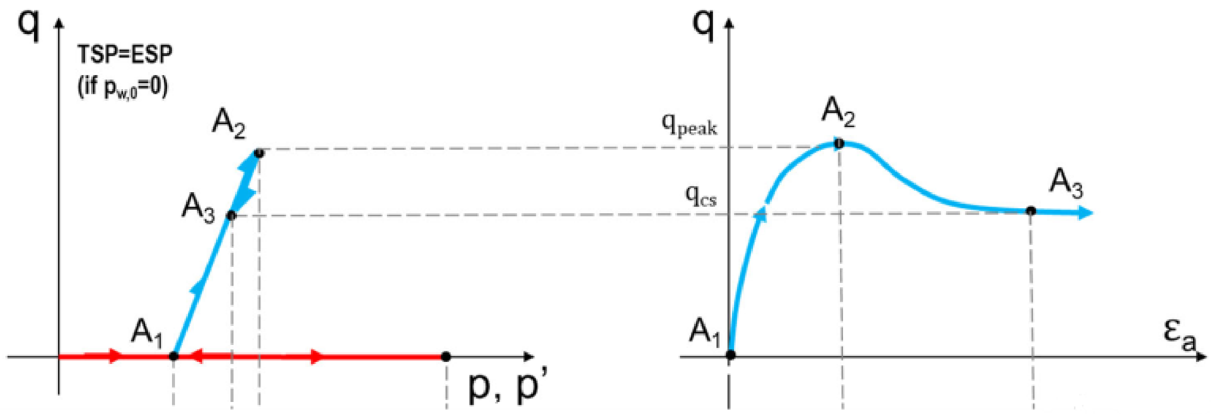
$$\varphi'_{cv} = \arcsin\left(\frac{3 \cdot M}{6 + M}\right) = \arcsin\left(\frac{3 \cdot 0.85}{6 + 0.85}\right) = 22°$$

- The critical state shear strength is independent of the initial condition of the soil, i.e. the initial void ratio.

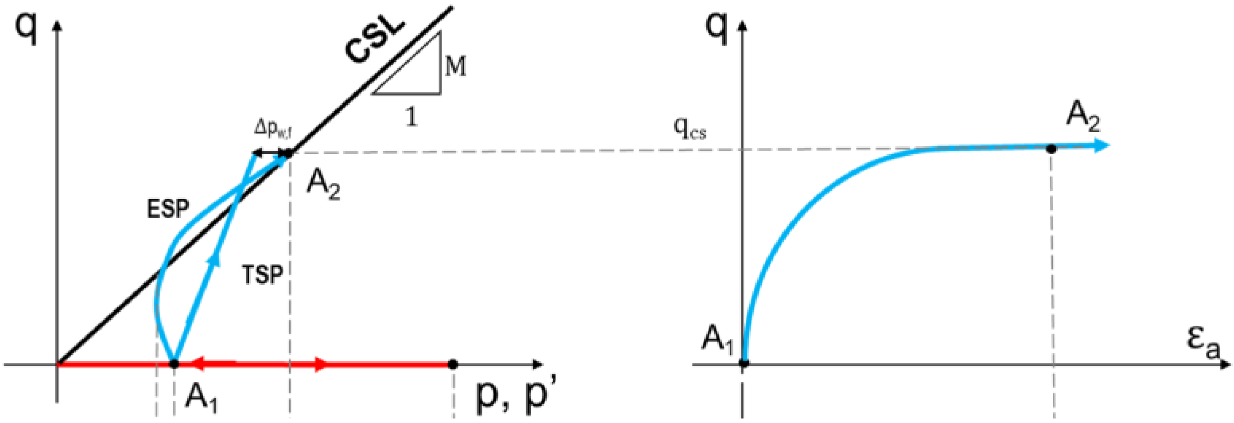
Question 3 – Overconsolidation

- Consider that samples in questions 1 and 2 are overconsolidated. Draw schematically $(q - p')$, $(q - p)$ and $(q - \varepsilon_a)$ graphs.

Drained condition:



Undrained:



- Will the critical state line change if the soil is overconsolidated? Explain why.
  No. Overconsolidation creates peak strength that is higher than critical one, however critical shear strength is unique for every type for soil so as the critical state line.

Annex

Computing the volumetric strain ε_v by knowing the initial void ratio e_0 and the void ratio variation Δe

$$e = \frac{V_v}{V_s}$$

$$V_v = eV_s$$

$$\varDelta V_v = \varDelta(eV_s) = e_0 V_{s0} + \varDelta e V_{s0} + e_0 \varDelta V_s + \varDelta e \varDelta V_s - e_0 V_{s0} = \varDelta e V_{s0} + e_0 \varDelta V_s + \varDelta e \varDelta V_s$$

Being $\varDelta V_s = 0$

$$\varDelta V_v = \varDelta e V_{s0} \qquad (1)$$

$$\varepsilon_v = -\frac{\varDelta V}{V_0} = -\frac{\varDelta(V_v+V_s)}{V_0} = -\frac{\varDelta V_v}{V_0} \qquad (2)$$

By replacing (1) in (2):

$$\varepsilon_v = -\frac{\varDelta e V_{s0}}{V_0} = -\frac{\varDelta e}{\frac{V_0}{V_{s0}}} = -\frac{\varDelta e}{\frac{V_{v0} + V_{s0}}{V_{s0}}} = -\frac{\varDelta e}{\frac{V_{v0} + V_{s0}}{V_{s0}}} = -\frac{\varDelta e}{1 + e_0}$$

Definition of specific volume

$$\nu = \frac{V}{V_s} = \frac{V_v + V_s}{V_s} = 1 + e$$

Computing the volumetric strain ε_v by knowing the initial specific volume v_0 and the specific volume variation Δv

Being:

$$\varepsilon_v = -\frac{\varDelta V_v}{V_0}$$

and:

$$\nu = \frac{V}{V_s} = \frac{V_v + V_s}{V_s} = 1 + e$$

we get:

$$\varepsilon_v = -\frac{\varDelta \nu}{\nu_0}$$